Harmonic Oscillations in Force Fields

Ali F. AbuTaha

Herndon, Virginia, 20271 USA
Tel-Fax:

July 29, 1992

The common practice of referring oscillations to the position of equilibrium has masked the most important features of harmonic motion in gravitational, electric and nuclear force fields. We show that oscillations in force fields are distinctly different from simple harmonic motion. We clarify how the distinct difference between the two motions lies in the forcing functions responsible for each type of oscillation. Whereas simple harmonic motion is strictly the result of an externally applied force-pulse, or impulse, of short duration, force-field-harmonic-motion is strictly the result of a series of force pulses supplied by the field itself. Field forces sustain oscillations in the fields, and it is therefore incorrect to cancel the effect of these forces by modeling symmetric motion about the equilibrium position.

PACS numbers: 03., 12., 21., 31., 32., 33.

Simple harmonic motion (SHM) is the universally accepted model for the simplest vibratory motion of particles and bodies. This model has been adopted for atomic and molecular oscillators, electric and electronic oscillations and mechanical vibrations. The SHM model is usually referred to the equilibrium position (Fig. 1) which leads to symmetry, simplicity and mathematical elegance. The same tactic has been used for oscillations in force fields. This stratagem cancels the effect of the field force itself, and it obscures the correct behavior of an important parameter of the oscillations - the amplitude, A. Careful examination shows that the SHM model is not representative of oscillations in force fields.

Consider the horizontal mass-spring (Fig. 2a), a typical SHM model. The standard equation of motion and the solution for this model are the familiar,

$$m\ddot{x} + kx = 0 \tag{1}$$

$$x(t) = \frac{v_o}{\omega_n} \sin \omega_n t + x_o \cos \omega_n t \tag{2}$$

where, m is the mass, k the spring constant, x_o and v_o are the initial position and velocity, respectively, $\ddot{x}$ is acceleration, and ω_n (= $\sqrt{k/m}$) is the natural frequency. After start-up, only the spring restoring force, -kx, is involved in the motion. The mass travels between <u>-x and +x</u>, and the spring restoring force ranges between <u>-kx and +kx</u>.

The SHM oscillator is set in motion by pulling the mass a distance, say x, from equilibrium and then releasing it; or, strictly, by a force pulse applied to the mass when at stable equilibrium. The latter is evident from the requirement for a finite initial velocity, v_o, in the solution, Eq. (2).

Unlike the horizontal model, a vertical mass-spring system undergoes initial static deflection when the mass is added. The static deflection is usually canceled by selecting the deflected position to be the equilibrium position (Fig. 2b). Thus, the effect of the gravitational force field is canceled, and the system is reduced to a SHM model. The motion of this vertical system becomes identical to that of the horizontal system of Fig. 2a. Here, the mass also travels between -x and +x, and the spring restoring force ranges between -kx and +kx. Again, only the spring restoring force is involved in this motion.

The same technique of referring harmonic motion to the equilibrium position has also been used for other motions. Imagine a tunnel through the earth (Fig. 2c). An object dropped into the tunnel oscillates indefinitely, in the absence of air friction, etc. This motion has also been considered a SHM. The SHM model is used extensively in acoustics, optics, mechanics, electrical and electronic circuits, and atomic and molecular systems. In these cases, the oscillation is referred to the equilibrium position, and the effect of the field force is not explicitly included in the solution.

We now examine the fundamental difference between harmonic oscillations in force fields and simple harmonic motion. The comparison between the two must be based on the forcing functions that act on each. We first disregard the tactic of "pulling" the mass, or the particle, from equilibrium and then "releasing" it, because this practice introduces artificial capacitance into the model.

Consider the vertical mass-spring again, where we now refer the coordinate system, as we must, to the undeflected, normal, or natural position of the spring. When the mass is released from this position, the weight is suddenly and continually felt by the spring, and oscillation is as shown in Fig. 3a. The gravitational force, or the weight, in this case is locally constant, and we designate it F_o. This is to say that the gravitational field force is applied to the system, "during" the motion, in a series of pulses, the sum of which adds up to the unit-step-function (Fig. 4a). At every subsequent return to the starting point, the same constant unit-step-forcing-function is applied suddenly and continually. The same description applies to the spring restoring force. The standard equation of motion and the solution for this model are also familiar,

$$m\ddot{x} + kx = F_o \qquad (3)$$

$$x(t) = \frac{F_o}{k}(1 - \cos \omega_n t) \qquad (4)$$

This is the true description of undamped oscillations in a (constant) force field environment. When $\omega_n t$ is equal to π, the amplitude (here, the displacement) is equal to $-2x$. This is to say that the effect of the applied force, F_o, is doubled. Remember that this force, F_o, produces the static deflection, x, when the force is applied gradually.

We then propose that the correct description of undamped harmonic motion in a force field is given by equations 3 and 4, and not by eqs. 1 and 2. The mass travels between 0 and -2x, and not between -x and +x; and the spring restoring force ranges between 0 and -2kx, and not between -kx and +kx.

It is true that horizontal and vertical mass-springs with similar parameters oscillate at the same natural frequency, even though the latter is in a force field while the former is not. Notwithstanding this common feature, we have clarified the important difference between the two situations. We look again at the horizontal mass-spring of Fig. 1a. Let us begin with the spring in the undeformed, or natural, position and then "imagine" that a horizontal gravitational force field is suddenly turned on (Fig. 3b). Here, the mass will travel between 0 and -2x, and the restoring force in the spring will range between 0 and -2kx.

We finally note that if a particle, or an oscillator, is located at an equilibrium position (say, in the tunnel's center in Fig. 2c, or the position of equilibrium in Fig. 2b), then no manipulation of the field itself or the spring restoring force can cause the particle to oscillate about, or move from, the equilibrium position. An oscillator in a force field will oscillate if, and only if, it is found at a distance from the equilibrium position. The method by which a particle is displaced from equilibrium is not relevant. For example, the particle can be pulled from equilibrium and then released, it may be driven from that location by a force-pulse, or the force field may be turned on suddenly, as in electric circuits, when the particle is at a distance from equilibrium. Oscillations in gravitational, electric, or nuclear force fields are principally driven by a series of force pulses supplied by the field itself, which, when integrated over the sum of the pulses, gives the doubling effect of the amplitude, A, for the undamped case when $\omega_n t = \pi$. Compare the forcing function for force-field-oscillations (Fig. 4a) with that for the simple harmonic motion (Fig. 4b).

The analysis and reasoning given above can be extended to include dissipative forces, where the damped oscillations must be referred to the normal, or undeflected position. The same considerations must also be made when using Lagrange's equation, the Hamilton-Jacobi equation, or other methods to determine the motion of particles, such as, the electron, in a force field. Equation 4 can be extended to derive expressions for the frequency or wavelength of the oscillations, or to calculate the amplitude of intensity, e.g., of the forces, momenta, energy, current, voltage, etc.

In this article and in refs. 1-3, we have presented basic concepts and models in "motion" which are indispensable tools to classical, quantum, and relativity physics. These new tools are essential for any attempt at unification theories. By understanding the true nature of the motion of elementary particles, exceptional advancements can be achieved in medical, pharmaceutical, chemical, and materials fields. We hope that capable scientists, mathematicians, engineers and, even, philosophers, will inquire into and develop the many vistas that the new concepts offer.

This research has been fully sponsored by the author.

References:

1- AbuTaha, A. F., "Validity of the General Theory of Relativity," Unpublished paper, July 1992.

2- AbuTaha, A. F., "Bouncing Harmonic Motion and the Compton Effect," Unpublished paper, July 1992.

3- AbuTaha, A. F., "Bouncing harmonic motion and oscillations in force fields," Unpublished paper, July 1992.

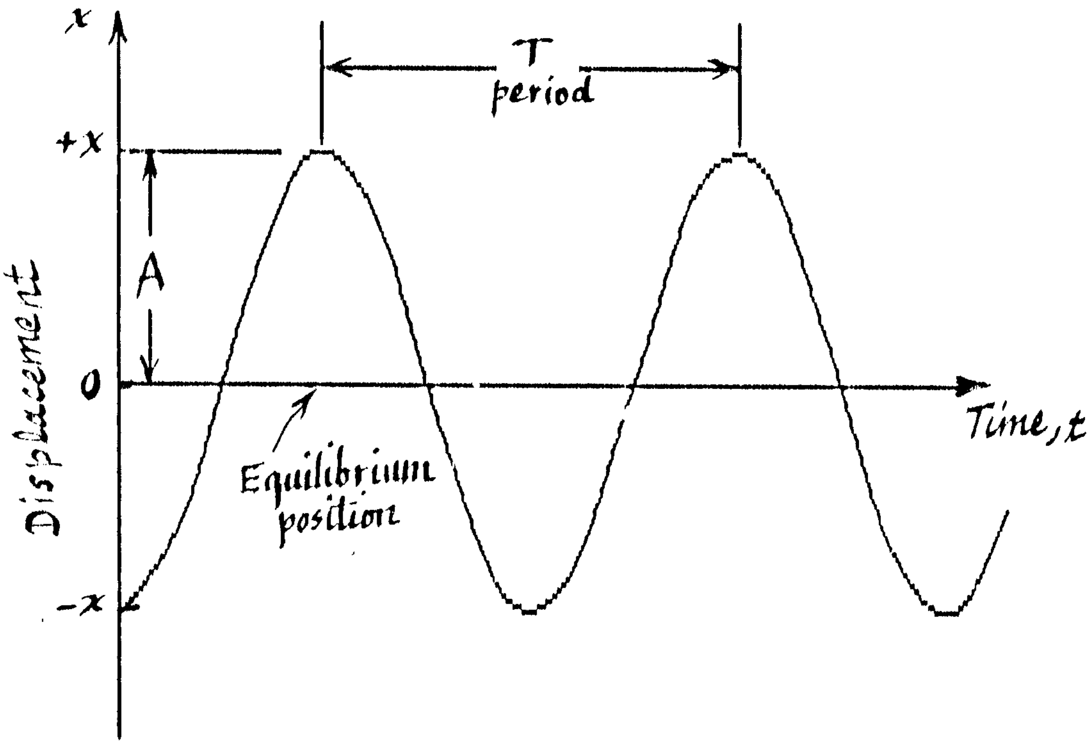


Fig. 1 Simple harmonic motion (SHM) of undamped oscillator about the equilibrium position.

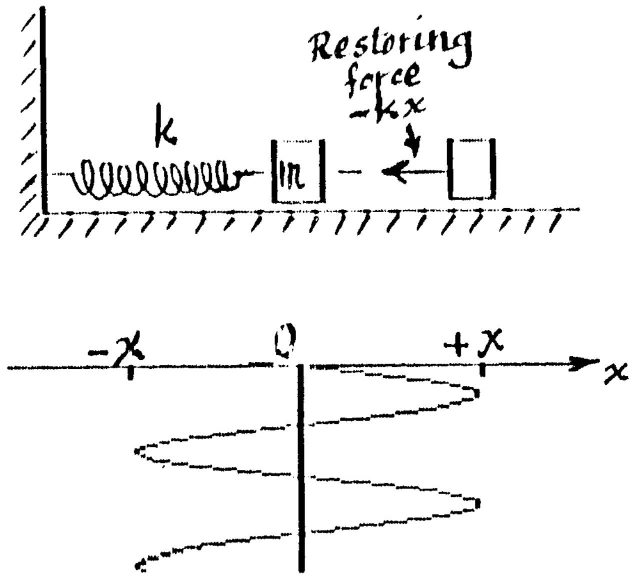


Fig. 2a Typical horizontal mass-spring SHM model oscillates about equilibrium.

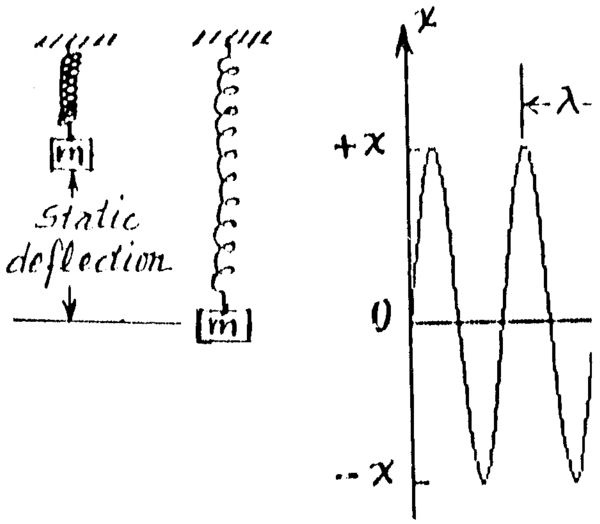


Fig. 2b Typical vertical mass-spring, made identical to SHM model by canceling the force field static deflection.

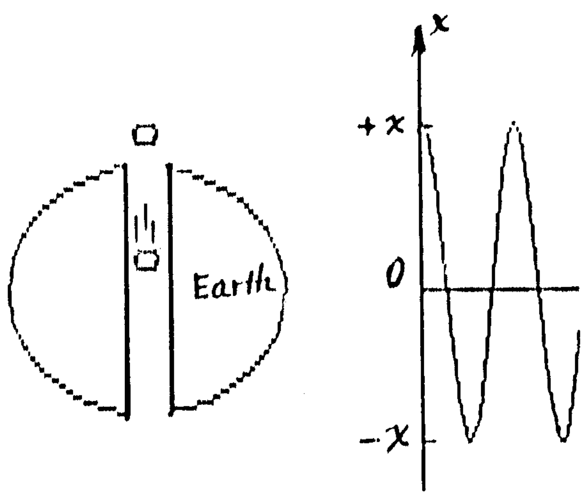
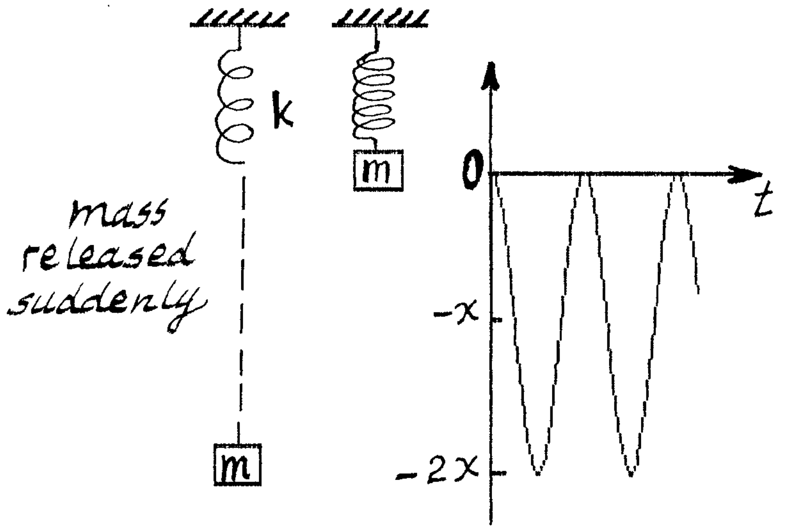
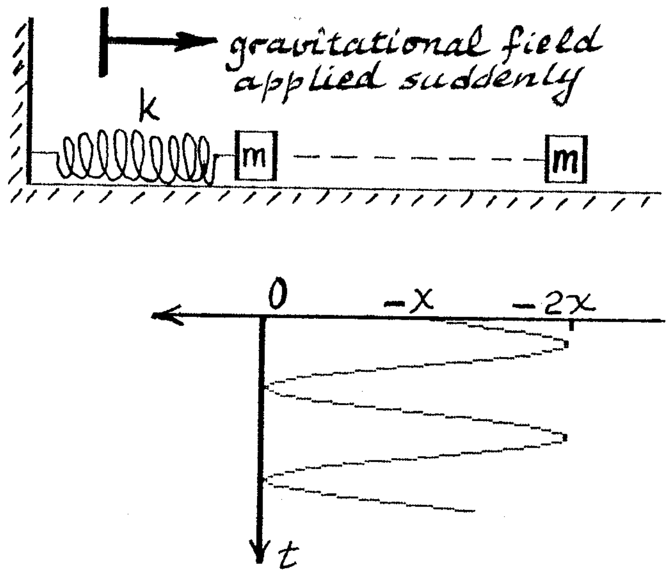


Fig. 2c Typical SHM model with oscillations taken about equilibrium position.



(a)



(b)

Fig. 3 The amplitude of oscillations in a force field is distinctly different from the SHM models. Notice the doubling of the amplitude.

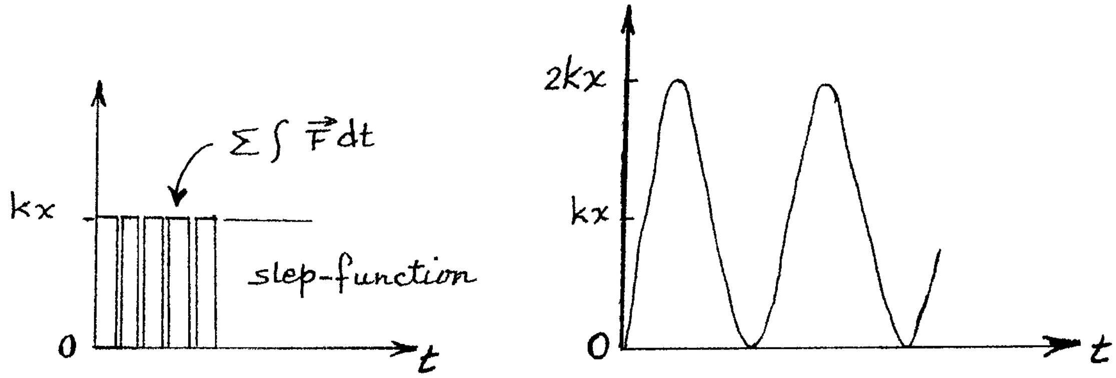


Fig. 4a Force-field-oscillations are driven by the field's own unit-step-forcing-function, or series of force-pulses; locally, with the magnitude of the field. Compare with SHM which is driven by one force-pulse.

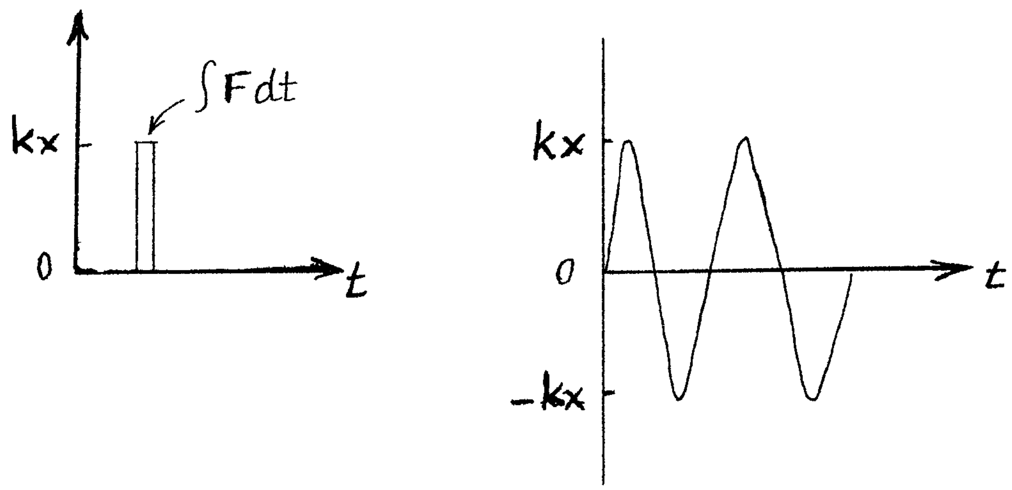


Fig. 4b The forcing function for simple harmonic motion (SHM) consists of one force-pulse which initiates the motion. Compare with force-field-oscillations which are driven by a series of force-pulses.