# Calculation of the Analemma - Part II

Gary D. Gordon April 2, 1994

This is a follow-up to my March 15 report, and Ali AbuTaha's comments of March 29. The latter report references the difference between the eccentric anomaly and true anomaly, and also the use of 2e in the equation for the actual sun's longitude, measured along the ecliptic as[1]

$$L_{sun} = L_{mean} + 1.915° \sin M_E + 0.020° \sin 2 M_E \qquad (6.30)$$

This is where the **eccentricity** of the earth's is used. The first coefficient is 2e (180°/$\pi$), and the second is (5/4) $e^2$ (180°/$\pi$). Your March 29 report is correct in focusing in on the difference between anomalies (angles) and on the "2e" factor. The objective of this report is to clarify these concepts.

The equation above includes an approximation for the solution of Kepler's equation. It does not imply a solution for an eccentricity of 2e, where the earth's orbit eccentricity (e = 0.016711). As was suggested in that paper, the approximation can be checked by calculating $M_E$ from $L_{sun}$; this is shown in the enclosed spreadsheet. The equations to do this were in the first report, and are listed below. The true anomaly v, the tangent of the eccentric anomaly tan E, and the mean anomaly $M_E$, are

$$v = M_E + L_{sun} - L_{mean}$$

$$\tan E = [(1 - e^2)^{`} \sin v]/(\cos v + e) \qquad \text{(Fig. 6.24)}$$

$$M_E = E - e \sin E \qquad (6.43)$$

If $M_E$ doesn't come out right, then $L_{sun}$ has to be adjusted until it does. The last equation is the famous Kepler's equation, which has to be solved numerically, or approximated by other methods (such as a series expansion). Comparing column d on p. 4 with column q on p. 5, $M_E$ does come out the same for the accuracy calculated. Note there is no "2e" in any of the above, exact, equations.

<u>First Table</u>, Columns a-i (p. 4)

This is almost the same as the table in the March 15 report. The dates chosen for calculation have been changed so that the mean anomaly M runs from 0 to 360°. Note that the last increment is only five days, instead of the usual ten.

The mean longitude $L_{mean}$ and the true longitude $L_{sun}$ (both measured along the ecliptic) are similar to the mean anomaly and

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[1] Equation numbers refer to W. L. Morgan and G. D. Gordon, **Handbook of Communications Satellites**, Wiley, 1989, pp. 799-809.

true anomaly. However, they are measured from the First Point of Aries rather than from perigee, so their zero occurs around Mar 23, instead of Jan 3. This explains the first equation above, since $L_{sun} - L_{mean} = v - M_E$. As explained in the first report, the anomaly is obtained by plotting column g vs. column i.

Second Table, Columns j-q (p. 5)

The first three columns (j, k, l) are the true anomaly, eccentric anomaly, and mean anomaly. These are calculated with the three equations on the previous page. The angles are expressed in radians rather than degrees, to emphasize the relation with orbit eccentricity. Note that all three start at zero (perigee), reach $\pi$ radians around July 7 (apogee), and then reach $2\pi$ at the following Jan 3 (perigee). At these actual points these values are 0, $\pi$, and $2\pi$, and are identical to each other. Around perigee the true anomaly changes faster than the other two, and the mean anomaly is the slowest. Around apogee the opposite is true.

The next three columns (m, n, o) show differences between the various anomalies (angles). The differences (v - E) and (E - M) are almost identical, and vary sinusoidal to ±e, or a total variation of 2e. This means that the eccentric anomaly E is halfway between the true anomaly v and the mean anomaly M.

The most important part of the table (column o) is the difference between the true anomaly and the mean anomaly, that is, (v - M). This also varies sinusoidal, but at twice the amplitude. It varies ±2e, or a total variation of 4e. Measured along the ecliptic, the actual sun can get 2e radians ahead or behind the mean sun. With a mean earth-sun distance of 150 million km (or 150 gigameters), the actual sun distance along the ecliptic varies by ±5 million km (for e = 0.016711).

For comparison, the variation in radial distance (r/a - 1) is listed in column p. This is calculated from

$$r = p/(1 + e \cos v)$$

$$r/a - 1 = (1 - e^2)/(1 + e \cos v) - 1$$

where r is the earth-sun distance, $p = a(1 - e^2)$ is the parameter, e the eccentricity, and v the true anomaly. Note that this varies sinusoidally by ±e around the mean, or ±2.5 million km.

The variation along the orbit of the actual sun around the mean sun is twice the radial variation. This factor of two is independent of the eccentricity of the orbit. The actual sun moves in an ellipse around the mean sun, with the major axis equal to twice the minor axis.

I hope this helps in understanding the factors of e and 2e.

| a | b | c | d | e | f | g | h | i |
|---|---|---|---|---|---|---|---|---|
| | | | ANALEMMA | | | | GDGordon | |
| | 94 | | | | | | | |
| d | Date | L mean | M | L sun | alph | decl | GHA | 4*long |
| (day) | | (deg) | (deg) | (deg) | (deg) | (deg) | (deg) | ~~(deg)~~ (min) |
| -2189 | 03-Jan | 282.88 | 0.049 | 282.88 | -76.01 | -22.815 | 282.88 | 4.464 |
| -2179 | 13-Jan | 292.73 | 9.905 | 293.07 | -65.10 | -21.467 | 292.73 | 8.671 |
| -2169 | 23-Jan | 302.59 | 19.761 | 303.25 | -54.45 | -19.430 | 302.59 | 11.837 |
| -2159 | 02-Feb | 312.45 | 29.617 | 313.41 | -44.12 | -16.796 | 312.45 | 13.718 |
| -2149 | 12-Feb | 322.30 | 39.473 | 323.54 | -34.13 | -13.673 | 322.30 | 14.251 |
| -2139 | 22-Feb | 332.16 | 49.329 | 333.63 | -24.46 | -10.175 | 332.16 | 13.534 |
| -2129 | 04-Mar | 342.02 | 59.185 | 343.68 | -15.04 | -6.418 | 342.02 | 11.781 |
| -2119 | 14-Mar | 351.87 | 69.041 | 353.67 | -5.81 | -2.512 | 351.87 | 9.278 |
| -2109 | 24-Mar | 1.73 | 78.897 | 3.62 | 3.32 | 1.438 | 1.73 | 6.355 |
| -2099 | 03-Apr | 11.59 | 88.753 | 13.50 | 12.42 | 5.329 | 11.59 | 3.348 |
| -2089 | 13-Apr | 21.44 | 98.609 | 23.33 | 21.59 | 9.063 | 21.44 | 0.584 |
| -2079 | 23-Apr | 31.30 | 108.465 | 33.10 | 30.89 | 12.547 | 31.30 | -1.650 |
| -2069 | 03-May | 41.16 | 118.321 | 42.82 | 40.38 | 15.687 | 41.16 | -3.121 |
| -2059 | 13-May | 51.01 | 128.177 | 52.50 | 50.09 | 18.395 | 51.01 | -3.686 |
| -2049 | 23-May | 60.87 | 138.033 | 62.13 | 60.04 | 20.587 | 60.87 | -3.309 |
| -2039 | 02-Jun | 70.72 | 147.889 | 71.72 | 70.20 | 22.192 | 70.73 | -2.084 |
| -2029 | 12-Jun | 80.58 | 157.745 | 81.29 | 80.52 | 23.153 | 80.58 | -0.234 |
| -2019 | 22-Jun | 90.44 | 167.601 | 90.84 | 90.92 | 23.436 | 90.44 | 1.912 |
| -2009 | 02-Jul | 100.29 | 177.457 | 100.38 | 101.29 | 23.033 | 100.29 | 3.973 |
| -1999 | 12-Jul | 110.15 | 187.313 | 109.91 | 111.54 | 21.962 | 110.15 | 5.574 |
| -1989 | 22-Jul | 120.01 | 197.169 | 119.45 | 121.61 | 20.265 | 120.01 | 6.416 |
| -1979 | 01-Aug | 129.86 | 207.025 | 129.01 | 131.44 | 18.004 | 129.86 | 6.312 |
| -1969 | 11-Aug | 139.72 | 216.881 | 138.59 | 141.02 | 15.254 | 139.72 | 5.205 |
| -1959 | 21-Aug | 149.58 | 226.737 | 148.20 | 150.37 | 12.099 | 149.58 | 3.164 |
| -1949 | 31-Aug | 159.43 | 236.593 | 157.85 | 159.52 | 8.624 | 159.43 | 0.358 |
| -1939 | 10-Sep | 169.29 | 246.449 | 167.55 | 168.55 | 4.920 | 169.29 | -2.968 |
| -1929 | 20-Sep | 179.15 | 256.305 | 177.29 | 177.52 | 1.076 | 179.15 | -6.515 |
| -1919 | 30-Sep | 189.00 | 266.161 | 187.09 | -173.49 | -2.816 | 189.00 | -9.954 |
| -1909 | 10-Oct | 198.86 | 276.017 | 196.95 | -164.38 | -6.660 | 198.86 | -12.946 |
| -1899 | 20-Oct | 208.72 | 285.873 | 206.86 | -155.07 | -10.354 | 208.72 | -15.162 |
| -1889 | 30-Oct | 218.57 | 295.729 | 216.83 | -145.51 | -13.795 | 218.57 | -16.310 |
| -1879 | 09-Nov | 228.43 | 305.585 | 226.85 | -135.61 | -16.871 | 228.43 | -16.170 |
| -1869 | 19-Nov | 238.29 | 315.441 | 236.92 | -125.37 | -19.469 | 238.29 | -14.632 |
| -1859 | 29-Nov | 248.14 | 325.297 | 247.03 | -114.79 | -21.484 | 248.14 | -11.740 |
| -1849 | 09-Dec | 258.00 | 335.153 | 257.18 | -103.93 | -22.821 | 258.00 | -7.721 |
| -1839 | 19-Dec | 267.85 | 345.009 | 267.35 | -92.89 | -23.412 | 267.85 | -2.975 |
| -1829 | 29-Dec | 277.71 | 354.865 | 277.54 | -81.80 | -23.225 | 277.71 | 1.974 |
| -1824 | 03-Jan | 282.64 | 359.793 | 282.63 | -76.27 | -22.839 | 282.64 | 4.350 |

```
A:A7: -2189
A:B7: (D2) [W7] 36526+A7
A:C7: (F2) [W8] @MOD(280.46+0.9856474*A7,360)+360
A:D7: (F3) [W8] @MOD(357.528+0.9856003*A7,360)+360
A:E7: (F2) [W8] +C7+1.915*@SIN(D7*@PI/180)+0.02*@SIN(2*D7*@PI/180)
A:F7: (F2) @ATAN2(@COS(@PI*E7/180),@COS(@PI*23.439/180)*@SIN(@PI*E7/180))*(180/π)
A:G7: (F3) [W8] (180/@PI)*@ATAN(@TAN(@PI*23.439/180)*@SIN(@PI*F7/180))
A:H7: (F2) [W7] @MOD(100.4602346+0.985647348*(A7-0.5)+15.041068*12,360)+360
A:I7: (F3) [W8] @IF(F7-H7<-180,F7-H7+360,F7-H7)*4
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| j | k | l | m | n | o | p | q |
|---|---|---|---|---|---|---|---|
| | | | ANALEMMA. | ECC = 0.016711 | | | |
| True Anomaly | Eccentric Anomaly | Mean Anomaly | | | | | GDGordon |
| v (rad) | E (rad) | M (rad) | v - E (rad) | E - M (rad) | v - M (rad) | r/a - 1 | M (deg) |
| 0.0009 | 0.0009 | 0.0009 | 0.0000 | 0.0000 | 0.0000 | -0.0167 | 0.049 |
| 0.1787 | 0.1758 | 0.1729 | 0.0029 | 0.0029 | 0.0059 | -0.0165 | 9.905 |
| 0.3564 | 0.3506 | 0.3449 | 0.0058 | 0.0057 | 0.0115 | -0.0157 | 19.761 |
| 0.5337 | 0.5253 | 0.5169 | 0.0084 | 0.0084 | 0.0168 | -0.0145 | 29.617 |
| 0.7105 | 0.6997 | 0.6889 | 0.0108 | 0.0108 | 0.0216 | -0.0128 | 39.473 |
| 0.8866 | 0.8738 | 0.8610 | 0.0129 | 0.0128 | 0.0257 | -0.0107 | 49.329 |
| 1.0620 | 1.0474 | 1.0330 | 0.0145 | 0.0145 | 0.0290 | -0.0084 | 59.185 |
| 1.2364 | 1.2207 | 1.2050 | 0.0157 | 0.0157 | 0.0314 | -0.0057 | 69.041 |
| 1.4099 | 1.3935 | 1.3770 | 0.0165 | 0.0164 | 0.0329 | -0.0029 | 78.897 |
| 1.5825 | 1.5657 | 1.5490 | 0.0167 | 0.0167 | 0.0334 | -0.0001 | 88.753 |
| 1.7540 | 1.7375 | 1.7211 | 0.0165 | 0.0165 | 0.0329 | 0.0028 | 98.609 |
| 1.9246 | 1.9088 | 1.8931 | 0.0157 | 0.0158 | 0.0315 | 0.0055 | 108.465 |
| 2.0942 | 2.0797 | 2.0651 | 0.0145 | 0.0146 | 0.0291 | 0.0081 | 118.321 |
| 2.2630 | 2.2501 | 2.2371 | 0.0129 | 0.0130 | 0.0259 | 0.0105 | 128.177 |
| 2.4311 | 2.4202 | 2.4091 | 0.0110 | 0.0110 | 0.0220 | 0.0125 | 138.033 |
| 2.5986 | 2.5899 | 2.5811 | 0.0087 | 0.0088 | 0.0175 | 0.0142 | 147.889 |
| 2.7656 | 2.7594 | 2.7532 | 0.0062 | 0.0062 | 0.0124 | 0.0155 | 157.745 |
| 2.9322 | 2.9287 | 2.9252 | 0.0035 | 0.0035 | 0.0070 | 0.0163 | 167.601 |
| 3.0987 | 3.0979 | 3.0972 | 0.0007 | 0.0007 | 0.0015 | 0.0167 | 177.457 |
| -3.0181 | -3.0160 | -3.0140 | -0.0021 | -0.0021 | -0.0042 | 0.0166 | 187.313 |
| -2.8516 | -2.8468 | -2.8419 | -0.0048 | -0.0049 | -0.0097 | 0.0160 | 197.169 |
| -2.6848 | -2.6774 | -2.6699 | -0.0074 | -0.0075 | -0.0149 | 0.0149 | 207.025 |
| -2.5176 | -2.5078 | -2.4979 | -0.0098 | -0.0099 | -0.0197 | 0.0135 | 216.881 |
| -2.3499 | -2.3379 | -2.3259 | -0.0120 | -0.0120 | -0.0240 | 0.0116 | 226.737 |
| -2.1814 | -2.1677 | -2.1539 | -0.0138 | -0.0138 | -0.0276 | 0.0094 | 236.593 |
| -2.0122 | -1.9971 | -1.9818 | -0.0152 | -0.0152 | -0.0304 | 0.0069 | 246.449 |
| -1.8421 | -1.8260 | -1.8098 | -0.0161 | -0.0162 | -0.0323 | 0.0042 | 256.305 |
| -1.6711 | -1.6545 | -1.6378 | -0.0166 | -0.0167 | -0.0333 | 0.0014 | 266.161 |
| -1.4991 | -1.4824 | -1.4658 | -0.0167 | -0.0166 | -0.0333 | -0.0015 | 276.017 |
| -1.3261 | -1.3099 | -1.2938 | -0.0162 | -0.0161 | -0.0323 | -0.0043 | 285.873 |
| -1.1521 | -1.1369 | -1.1217 | -0.0152 | -0.0152 | -0.0304 | -0.0070 | 295.729 |
| -0.9772 | -0.9634 | -0.9497 | -0.0138 | -0.0137 | -0.0275 | -0.0095 | 305.585 |
| -0.8015 | -0.7896 | -0.7777 | -0.0119 | -0.0119 | -0.0238 | -0.0118 | 315.441 |
| -0.6250 | -0.6153 | -0.6057 | -0.0097 | -0.0096 | -0.0194 | -0.0136 | 325.297 |
| -0.4480 | -0.4408 | -0.4337 | -0.0072 | -0.0071 | -0.0143 | -0.0151 | 335.153 |
| -0.2705 | -0.2660 | -0.2616 | -0.0044 | -0.0044 | -0.0088 | -0.0161 | 345.009 |
| -0.0927 | -0.0911 | -0.0896 | -0.0015 | -0.0015 | -0.0031 | -0.0166 | 354.865 |
| -0.0037 | -0.0037 | -0.0036 | -0.0001 | -0.0001 | -0.0001 | -0.0167 | 359.793 |

```
A:J7: (F4) (@IF(D7<180,D7,D7-360)+E7-C7)*@PI/180
A:K7: (F4) @ATAN2(@COS(J7)+$O$2,@SQRT(1-$O$2^2)*@SIN(J7))
A:L7: (F4) +K7-$O$2*@SIN(K7)
A:M7: (F4) +J7-K7
A:N7: (F4) +K7-L7
A:O7: (F4) +J7-L7
A:P7: (F4) (1-$O$2^2)/(1+$O$2*@COS(J7))-1
A:Q7: (F3) @IF(L7>0,L7,L7+2*@PI)*180/@PI
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COMMENTS (Part II) on
**The Cause of Gravity**

Gary D. Gordon April 6, 1994

Here are some comments on your responses, dated March 29 and March 31.

I was interested in your data on the accuracy of the Cavendish experiment. The results you quote show that the experiment is less accurate than what I had guessed. While you note a change, you didn't quote any temperature measurements. I would think a more quantitative measurement could be made by changing the room temperature and the entire apparatus. It should be possible to detect a 10% change due to a $\Delta T = 7°C$, or at least a 20% change due to a $\Delta T = 15°C$.

I think you are close to understanding the differences between the various calculations of the analemma. I hope the enclosed calculations (pp. 2-5) will help.

You discussed the difference between the satellite analemma and the Earth's analemma. An analemma can be drawn for any body orbiting around the earth. This is simply the declination (or latitude of subsatellite point) plotted against the difference between the true anomaly (actual position) and the mean anomaly. This difference is projected on the equatorial plane. This variable is the longitudinal difference between the actual satellite position and the mean position.

The uniqueness of the Earth's anomaly is that we are calculating the position of the sun in an earth-centered coordinate system. That is, we assume the sun is going around the earth. This is a useful assumption for a number of calculations.

The uniqueness of the geostationary satellite is that there is a point on earth equivalent to a mean position. Because the orbit period is the same as the earth's rotational motion, we can use the earth's surface to plot the analemma. Thus the trace of the subsatellite point on the earth is an analemma.

If the earth rotated once a year, then the sun would trace the earth's analemma on the earth. By plotting the position of the sub-solar point at 12:00 GMT every day, I slowed the earth's rotation as if I were illuminating it with a flash once a day. If we launched a satellite with an orbit period of one sidereal day, an inclination of 23.5°, and an eccentricity of 0.016711, the sub-satellite point would trace a curve on the earth's surface identical to the earth's analemma.



cc: A. Meulenberg