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My Dear Friend:

It was thoughtful of you to send pages of the "Explanatory Supplement" to the Astronomical Ephemeris. There is a fundamental and substantial discrepancy that is not obvious to the casual observer. I will explain.

The referenced expression for **normal gravity** in the Supplement, on page 169, is;

$$g = 978.049\ (1 + 0.00528\ 84 \sin^2 \phi - 0.00000\ 59 \sin^2 2\phi)\ \text{cm/sec}^2$$

The expression is basically similar to others I used, e.g.,

$$g = 978.0516[1 + 0.005291 \sin^2\phi - 0.0000057 \sin^2 2\phi + 0.000 \cos^2\phi \cos 2(\lambda + 6^\circ)]\ \text{cm/sec}^2$$

where, λ is the longitude. There are also more formidable formulas with more harmonic expansion terms. But, in all cases, only the first two terms are necessary for our purpose; or,

$$g = 978.049\ (1 + 0.00528\ 84 \sin^2 \phi)\ \text{cm/sec}^2$$

Using the above expression, we find g in the poles and the equator to be:

g_{pole}	$= 983.221$ cm/sec^2	Measured value [983.217]
$g_{equator}$	$= 978.049$ cm/sec^2	Measured value [978.039]

All would seem well, BUT IT IS NOT. The problem is not in comparing the Calculated and Measured accelerations in the poles and equator. That comparison conceals the real problem. The central problem is to explain the difference between g_{pole} and $g_{equator}$.

$$\Delta\ \text{(Calculated)} = 983.221 - 978.049 = 5.172\ \text{cm/sec}^2$$

$$\Delta\ \text{(Measured)} = 983.217 - 978.039 = 5.178\ \text{cm/sec}^2$$

The difference, Δ, is due to the different radii and the centrifugal acceleration at the equator.

From Page 169 of the same Explanatory Supplement: "The expression for gravity, in which ϕ is the geodetic latitude, ***includes the effect of centrifugal force due to the rotation of the Earth***." The ***centrifugal acceleration***, or $g_{cent.}$, is found from the angular speed of rotation and radius of the Earth, or from the period of rotation and radius of the Earth. Using the Supplement's values, $g_{cent.}$ is **3.373 cm/sec^2** (I rounded this to 3.4 in my report).

We now subtract the ***centrifugal acceleration*** from the accelerations calculated above, or,

g_{pole}	$= 978.049 - 0.000 = 978.049$ cm/sec^2
$g_{equator}$	$= 983.221 - 3.373 = 979.848$ cm/sec^2

Or, a Δg of **1.799 cm/sec^2** which is unaccounted for. This represents 1 part in 550.

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I have checked a variety of expressions and dimensions including, polar and equatorial radii, different mass or gravitational parameter values, measured Meridian lengths, measured equatorial lengths, etc. In all cases, there is 1 part in 200 to 1 part in 600 that remains unexplainable.

Alternatively, let us use the Astronomical Constants on page K5:

Geocentric gravitational constant = 398 603 x 10^9 m^3 s^{-2}

Equatorial primary radius = 6 378 160 m

Polar radius = 6 356 774.7 m

We simply calculate the accelerations g_{pole} and $g_{equator}$ from the inverse square law ($g = \mu/R^2$),

$$g_{pole} = \frac{398603 \times 10^9 \text{ m}^3 \text{ s}^{-2}}{(6{,}356{,}774.7 \text{ m})^2} = 986.432 \text{ cm/sec}^2$$

$$g_{equator} = \frac{398603 \times 10^9 \text{ m}^3 \text{ s}^{-2}}{(6{,}378{,}160 \text{ m})^2} = 979.828 \text{ cm/sec}^2$$

$$\Delta = 6.604 \text{ cm/sec}^2$$

$$g_{cent} = 3.373 \text{ cm/sec}^2$$

$$\Delta g = 3.231 \text{ cm/sec}^2$$

Which represents a discrepancy of 1 part in 305.

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Enclosures